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Transition Onset Prediction for High-Speed Flow

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An approach that treats nonturbulent fluctuations in a turbulencelike manner and determines onset and extent of transition as part of a flow calculation is used to study transition at high Mach numbers. Three sets of previously obtained experimental data involving straight and flared cones at zero angle of attack are used to calibrate and validate the model. Two sets, at Mach numbers 3.5 and 6, were carried out in quiet tunnels, and the third set was carried out at a Mach number of 8 in a conventional tunnel. The results suggest that the second mode is not the only mode responsible for transition at high Mach numbers. In general, fair to good agreement with measured recovery factors, adiabatic wall temperatures, and heat-transfer rates is indicated.

Nomenclature

a = model constants (first mode)
b = model constants (second mode)
C_μ = constant, 0.09
k = fluctuating kinetic energy, m^2/s^2
M = Mach number
$\tilde{m}$ = rms of mass flux, $kg/m^2 \cdot s$
$\bar{m}$ = mean mass flux, $kg/m^2 \cdot s$
P = pressure, Pa
Q = mean velocity
q = heat flux at the wall, W/m^2
Re = Reynolds number
R_T = parameter defined in Eq. (18)
r = recovery factor
s_{ij} = strain tensor, s^{-1}
T = temperature, K
Tu = turbulent intensity
U_p = phase velocity, m/s
$\tilde{u}_i$ = mean velocity vector, m/s
x_t = location of transition onset (m)
α = parameter defined in Eq. (17)
Γ = intermittency
δ = boundary-layer thickness, m
δ^* = displacement thickness, m
ζ = enstrophy, s^{-2}
λ = wavelength, m
μ = viscosity, $kg/m \cdot s$
ν = kinematic viscosity, m^2/s
ξ = parameter defined in Eq. (16)
ρ = density, kg/m^3
τ = timescale, s
τ_k = decay time, s
ψ = direction of wave number, deg
ω = frequency, s^{-1}

Subscripts

aw = adiabatic wall
e = boundary-layer edge
nt = nonturbulent
t = turbulent
w = wall condition
0 = stagnation condition

Introduction

THE study of high-speed transition is very important for the efficient design of hypersonic vehicles such as the X-33, X-34, and other reusable launch vehicles. Important design parameters, such as skin friction and heat transfer, are increasing rapidly or are maximum within the transitional region. The inability to predict accurately transition onset and extent on hypersonic vehicles has been an impediment to designing low-cost space transportation systems.

The purpose of this investigation is to address this area of research by adapting a newly developed approach for transitional onset prediction[1] to high-speed flows. The approach of Ref. 1 treats transitional flows in a turbulence-like manner. It is based on the fact that exact equations governing fluctuations are the same irrespective of their nature. To implement this approach, it is necessary to model correlations in the equations that govern the fluctuations in such a way so as not to invoke the nature of fluctuations. This modeling is accomplished by formulating a two-equation model that governs the variance of velocity (i.e., the kinetic energy of the fluctuations k) and the variance of vorticity (i.e., enstrophy ζ). To date, the relationship between the transition/turbulent stress and the strain rate has been described by the Boussinesq approximation, with differences in transitional and turbulent flows relating to eddy viscosity and fluctuation timescales. For turbulent flows traditional concepts have been used,[2,3] whereas for nonturbulent fluctuations scaling arguments based on linear stability theory have been employed.[1,4–9] As different scaling arguments can hold for different types of transition processes, the current method is not mechanism-independent, such as is the case for empirical correlations. Rather, it is dictated by the dominant mode of instability growth and the manner in which it is influenced by environmental factors such as freestream fluctuation intensity and surface roughness. This information can be gleaned from linear stability theory or experiment.[10]

The bridging of the nonturbulent and turbulent solutions is currently accomplished by the Dhawan–Narasimha intermittency expression.[11] This bridging requires the specification of a transition point that can be selected as the streamwise location at which skin friction, heat transfer, or recovery factor first attains a local

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minimum. Using this basic idea, successful simulations of transitional flows driven by Tollmein–Schlichting, crossflow, or free shear-layer instabilities have been conducted.[1,4–9] This has been accomplished using available computational fluid dynamics (CFD) codes and without having to use stability codes.

In the present work three different sets of experiments involving flared and straight cones were utilized to validate the approach. The first set of data was obtained by Chen et al.[12] in the NASA Langley Research Center Mach 3.5 Pilot Low-Disturbance Tunnel, using a 5-deg half-angle sharp straight cone. The second set is that of Lachowicz et al.[13,14] and Blanchard and Selby[15] conducted in the NASA Langley Research Center Mach 6 Quiet Tunnel[16] using different 5-deg sharp flared cones. The last set is that of Kimmel[17] carried out in the Arnold Engineering Development Center (AEDC) Tunnel B using straight and flared cones.

Formulation of the Problem

Approach

The model to be used in this study is an extension of the transition/turbulence model,[1] which treats nonturbulent fluctuations in a turbulence like manner. Traditionally, the transition problem has been treated as a combination of two problems. The first deals with the transition extent given the onset, whereas the second deals with the transition onset. One of the methods employed in calculating the extent is to replace the turbulent viscosity by

$$\Gamma\mu_t \tag{1}$$

where Γ is the fraction of the time the flow is turbulent at a given location. Γ varies from 0, at onset, to 1 when the transition to turbulence is complete. The most widely used expression for Γ is that of Dhawan and Narasimha.[11] There are many methods that can be used to specify transition onset: selection based on experiment, experimental correlation, or use of stability theory. Methods based on stability theory employ the e^n method or a method based on the parabolized stability equations (PSE).

Even when transition onset is specified from experimental results, Eq. (1) does not perform well. One of the reasons for this behavior is because the preceding formula does not account for the nonturbulent fluctuations that eventually lead to transition. This result led Young et al.[18] and Warren et al.[19] to replace μ_t by

$$(1-\Gamma)\mu_{\rm nt} + \Gamma\mu_t \tag{2}$$

where $\mu_{\rm nt}$ represents the contribution of the nonturbulent fluctuations and is derived using results from linear stability theory. In the work of Refs. 18 and 19, the onset was assumed.

In the present investigation the constitutive stress-strain relations for the nonturbulent fluctuations are derived from observed or computed characteristics of first- and second-mode instabilities. Because mechanisms responsible for transition are different for the two types of instabilities, corresponding stress-strain relations are different. In both cases, however, $\mu_{\rm nt}$ is set as

$$\mu_{\rm nt} = C_\mu \rho k \tau_{\rm nt}, \qquad C_\mu = 0.09 \tag{3}$$

The turbulence model is based on the k–ζ model of Robinson and Hassan.[3] This model is free of damping and wall functions and is coordinate independent. Further, all modeled correlations are tensorially consistent and Galilean invariant. The model reproduces the correct growth rates of all free shear layers and is capable of predicting separated flow in the presence and absence of shocks. The model also incorporates relevant compressibility effects and has been demonstrated for both two- and three-dimensional separated flows where Morkovin's hypothesis is expected to hold.

Transition from First-Mode Disturbances

For moderate supersonic Mach numbers, below approximately four, the dominant mode of instability is the oblique first mode. First-mode disturbances are vorticity disturbances that are characterized by fluctuations in velocity. Two-dimensional first-mode disturbances in low-speed flows are known as Tollmien–Schlichting waves. For this case τ is chosen as[1]

$$\tau_1 = a/\omega \tag{4}$$

where a is a model constant that depends on the freestream intensity Tu defined as

$$Tu = 100\sqrt{\tfrac{2}{3}\left(k_\infty \mid Q_\infty^2\right)} \tag{5}$$

where k_∞ and Q_∞ are the freestream kinetic energy of the fluctuations and magnitude of freestream velocity. The model constant has the form[1]

$$a = 0.069(Tu - 0.138)^2 + 0.00819 \tag{6}$$

The quantity ω is the frequency of the first-mode disturbance having the maximum amplification rate and is given by a correlation developed by Walker[20] as

$$\omega\nu/Q_e^2 = 3.2Re_{\delta^*}^{-\frac{3}{2}} \tag{7}$$

where Re_{δ^*} is the edge Reynolds number based on the displacement thickness δ^*. The preceding correlation is based on results of stability calculations for Falkner–Skan profiles obtained by Obremski et al.[21] and is suited for flows where the pressure gradient is not necessarily zero.

For high-speed flows Eqs. (4) and (7) are modified in two different ways. The first is to allow for compressibility effects. Following Warren et al.,[18] a reference temperature T^* determined as

$$T^*/T_e = 1 + 0.032M_e^2 + 0.56(T_w/T_e - 1) \tag{8}$$

is used to calculate ν in Eq. (7). The second modification is to allow for the oblique modes, which is accomplished in this work by dividing the expression in Eq. (4) by $(\cos\psi)^{1/2}$, where ψ is the angle between the wave-number vector and the freestream. This choice reduces to the two-dimensional mode when $\psi = 0$ and was arrived at by numerical optimization. Estimates of ψ are taken from Fig. 10.11 of Ref. 22.

Second-Mode Disturbances

At Mach numbers higher than four, the transition process is dominated by second-mode disturbances. Second-mode disturbances are acoustical disturbances characterized by large fluctuations in pressure and temperature and are much larger than velocity fluctuations. As the boundary-layer edge Mach number increases above 2.2, a region of the boundary layer becomes supersonic relative to the phase velocity. Mack[22] showed that when such a situation exists multiple solutions to the inviscid stability equation arise. These additional solutions are called the higher modes. The second mode is the most unstable of these higher modes. For this mode τ is chosen as

$$\tau_2 = b/\omega_{\rm sm} \tag{9}$$

with

$$\omega_{\rm sm} \simeq U_p/\lambda \tag{10}$$

where U_p is about 0.94 times the edge velocity. λ is approximately 2δ.

The total transitional contribution to the viscosity timescale is set as the sum of the contributions from the first and second modes:

$$\tau_{\rm nt} = \tau_1 + \tau_2 \tag{11}$$

Similarly, the decay time τ_k (Ref. 1) for the turbulent kinetic energy is modeled using contribution from nonturbulent and turbulent fluctuation as

$$1/\tau_k = (1-\Gamma)/\tau_{k,\rm nt} + \Gamma/\tau_{k,t} \tag{12}$$

$$1/\tau_{k,\rm nt} = (a+b)(\nu_t/\nu)s, \qquad 1/\tau_{k,t} = k/\nu\zeta \tag{13}$$

where

$$s^2 = \tilde{s}_{ij}\tilde{s}_{ij}, \qquad \tilde{s}_{ij} = \left(\frac{1}{2}\right)\left(\frac{\partial\tilde{u}_i}{\partial x_j} + \frac{\partial\tilde{u}_j}{\partial x_i}\right) \tag{14}$$

Intermittency and Onset Prediction Criteria

The intermittency Γ is given by Dhawan and Narasimha's expression[11] as

$$\Gamma(x) = 1 - \exp(-0.412\xi^2) \tag{15}$$

with

$$\xi = \max(x - x_t, 0)/\alpha \tag{16}$$

and α is a characteristic extent of the transitional region. An experimental correlation (20) between α and x_t is

$$Re_\alpha = 9.0 Re_{x_t}^{0.75} \tag{17}$$

with x_t being the location where turbulent spots first appear, or the onset of transition. In this work onset is determined as part of the solution.

A number of criteria may be used to determine transition onset.[1] The criteria can be one of the following: minimum skin friction, minimum heat transfer, or minimum recovery factor (adiabatic walls). Further, it was shown in Ref. 1 that a transition onset criterion based on the nonturbulent eddy viscosity correlates well with minimum skin-friction criterion for flat plates and airfoils. According to this criterion, onset is determined by the requirement

$$R_T = (1/C_\mu)(\nu_{\rm nt}/\nu) = 1 \tag{18}$$

The need to develop a criterion like this was a result of the fact that during the transient phase of a Navier–Stokes calculation surface derivatives have a tendency to fluctuate, and this fluctuation can delay or prevent convergence. It was found in Ref. 1 that the expression indicated in Eq. (18) was more forgiving and, in addition, gave accurate results.

The criteria of minimum skin friction and minimum heat transfer are valid for simple shapes such as flat plates and cones in the absence of adverse pressure gradient. For supersonic flow wall skin friction increases in the presence of adverse pressure gradients.[23] Similar behavior is observed for heat transfer. Thus, if one considers flared cones, such as the case for $M = 6$ experiments, then the criterion of minimum skin friction will not yield meaningful results if the transition takes place on the curved surface of the cone. In this case a criterion such as that indicated in Eq. (18) is more meaningful. On the other hand, if the transition takes place on the straight part of a flared cone such as was the case for $M = 8$, then minimum heat-transfer criterion is meaningful.

Model Constants

Intensity is not measured at hypersonic Mach numbers. Instead, the rms of mass flux $\tilde{m}$ over the mean mass flux $\bar{m}$ is measured. For low-speed flows this ratio reduces to the intensity. Thus, for present purposes $\tilde{m}/\bar{m}$ is a measure of the intensity.

The model constant appearing in Eq. (6) was correlated in Ref. 1 for flows where natural transition is expected to take place. Thus, the correlation is suited for studying flows in quiet tunnels such as the NASA Langley Research Center $M = 3.5$ and $M = 6$ tunnels. The AEDC Tunnel B is not a quiet tunnel in terms of freestream intensity levels. Donaldson and Coulter[24] reported values of intensity ranging from 1.5 to 3.8% for $M = 8$ and a range of Reynolds numbers. Because only a small portion of the spectrum is in the second-mode frequency range (frequencies more than 80–100 kHz), Stetson and Kimmel[25] suggested that a conventional hypersonic wind tunnel can be quiet for second-mode disturbances and noisy for first-mode disturbances.

The $M = 3.5$ experiments are dominated by oblique first mode, and values of $b \leq 3a$ had little or no effects on the calculated recovery factors and transition onset. Based on results of linear stability theory, the experiments at $M = 6$ and 8 are dominated by the second mode. The model constant b for the $M = 6$ experiments was correlated in terms of the model constant a using the onset criterion indicated in Eq. (18). Because of the high intensity levels in Tunnel B, it is not possible to relate b to the constant a that was developed for low intensity levels. Thus, both a and b for the $M = 8$ data were correlated by comparing with the data of Ref. 17 using a minimum heat flux criterion. Because of the different onset criteria employed, the resulting a and b values are not the same for the two sets of hypersonic Mach numbers.

Numerical Method

The modeling proposed here has been incorporated into Olynick and Hassan's[26] two-dimensional/axisymmetric implicit solver for hypersonic flows. This algorithm solves the governing equations for five-species air in thermochemical nonequilibrium. The solver uses Roe's flux-difference splitting[27] for the inviscid flux, extended to higher order using MUSCL variable extrapolation[28] with a minmod slope limiter. Time integration is accomplished using the diagonal implicit variant of Yoon and Jameson's lower-upper symmetric Gauss–Seidel method,[29] which only requires the inversion of diagonal matrices. This feature is attractive for nonequilibrium flows, where a large number of partial differential equations must be solved.

Because the current flowfields under investigation are approximately perfect-gas flows, vibrational relaxation and chemical reactions are disabled to obtain the results in this study. Additionally, the high-temperature transport-property calculations originally used in the code were replaced with Sutherland's law for viscosity and a constant laminar Prandtl number of 0.72. Closure for the Favre-averaged energy equation is accomplished using a constant turbulent Prandtl number of 0.89.

Results and Discussion

To validate the current approach, three test cases involving straight and flared cones were considered. Two cases were conducted in quiet tunnels, whereas the third was conducted in a conventional hypersonic tunnel. None of the experiments measured onset or extent of transition, they were inferred from heat transfer and recovery factor measurements. Thus, predictions of the theory are compared with recovery factors for the $M = 3.5$ test cases, with adiabatic wall temperatures for the $M = 6$ cases and with heat-transfer rates for the $M = 8$ cases. All cases considered assume zero angle of attack.

Because the theory of Ref. 1 and current extension are suited for the study of natural transition, model constants suited for the $M = 3.5$ and 6 experiments are not expected to yield correct transition onset predictions in conventional hypersonic tunnels. This is because the correlation for a indicated in Eq. (6) is limited to values of $Tu \leq 0.5$.

Two of the sets of experiments were the subject of an earlier investigation in Ref. 19. Grids as high as 275×75 (along and normal to the surface) were investigated there and here. Grids used in the results presented here are indicated next for each test case considered.

$M = 3.5$ Test Cases

The measurements of Chen et al.[12] were carried out in the NASA Langley Research Center Mach 3.5 Pilot Low-Disturbance Tunnel using 5-deg half-angle cones. Three cases from these experiments were selected for this investigation. All of the cases have the same Mach number but different stagnation pressures resulting in the following unit Reynolds numbers:

$$\text{Flow 3, case 5, } Re/L = 7.8 \times 10^7/\text{m}$$

$$\text{Flow 3, case 6, } Re/L = 5.89 \times 10^7/\text{m}$$

$$\text{Flow 3, case 7, } Re/L = 3.85 \times 10^7/\text{m}$$

The boundary-layer conditions for all cases were $M_e = 3.36$, $T_e = 98$ K, and $Tu = 0.1$, as reported by Singer et al.[30]

Figures 1–3 compare the prediction of the theory with experiment. The grid employed is 252×75. For these calculations the first-mode contribution to the nonturbulent timescale is given by

$$\tau_1 = a/\omega \cdot (\cos\psi)^{\frac{1}{2}} \tag{19}$$

with ψ, the angle between the wave-number vector and the freestream, determined from Fig. 10.11 of Ref. 22 as 63 deg. One

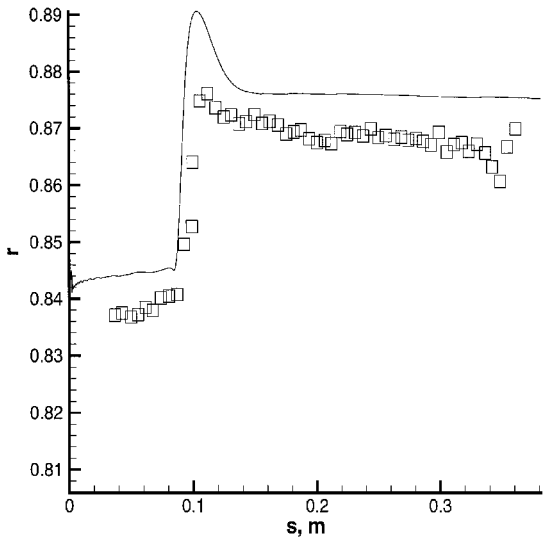

Fig. 1 Measured and computed recovery factor: M = 3.5 and Re/L = 7.8 × 10⁷/m.

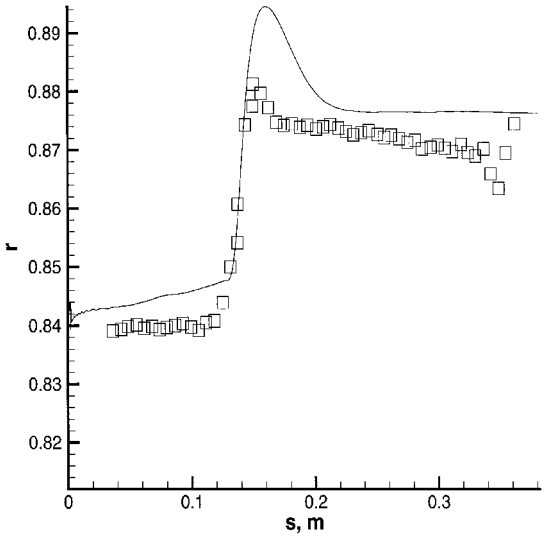

Fig. 2 Measured and computed recovery factor: M = 3.5 and Re/L = 5.89 × 10⁷/m.

recovers the normal mode expression given by Eq. (4) when $\psi = 0$. Onset is determined by the minimum skin-friction criterion. In general, good agreement with experiment is indicated. The contribution of the second mode was investigated. For

$$b \leq 3a \tag{20}$$

the second mode had minimal effects on the results shown in Figs. 1–3. The measured recovery factors in the turbulent region are lower than indicated by an earlier measurement by Mack[31] over cones at Mach numbers varying from 1.2 to 6.0 (Fig. 7.25 in Ref. 32).

M = 6 Test Cases

The usable quiet core length in the $M = 6$ tunnel was about 0.63 m (25 in.). This necessitated the use of flared cones to produce an adverse pressure gradient in order to produce measurable disturbance growth in the boundary layer. The flare design resulted in an approximately constant boundary-layer thickness and allowed the second-mode disturbance to grow. Two cones were tested in this facility. The first[13,14] was a 0.508-m (20-in.), 5-deg half-angle cone with the straight surface extending 0.254 m (10 in.); the flared surface has a radius of curvature of 2.36 m (93.07 in.). Transition was observed on the sharp version of the cone, and this is the geometry employed in this study. The second cone[15] was a 0.457-m (18-in.), 5-deg half-angle cone with the straight section extending 0.152 m

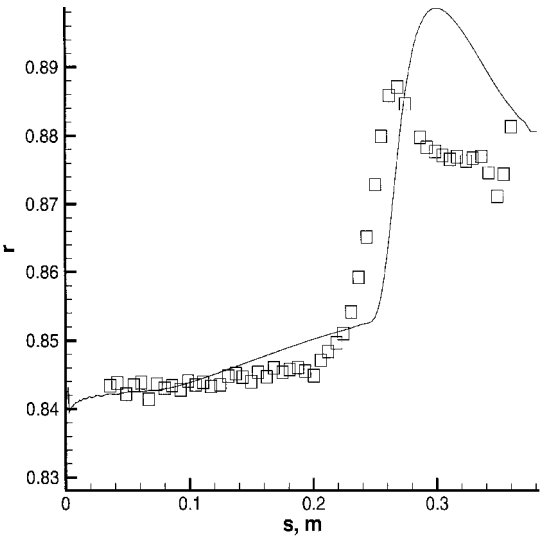

Fig. 3 Measured and computed recovery factor: M = 3.5 and Re/L = 3.85 × 10⁷/m.

(6 in.) with a flare radius of 2.34 m (91.94 in.). Only the adiabatic case was considered for this geometry. In both cases the stagnation pressure P_0 and temperature T_0 were 896.32 kPa (130 psia) and 450 K (810°R), respectively. In both investigations second-mode instability was deemed to be that responsible for transition.

Both experiments presented a major challenge because of the apparent inconsistency in the experimental data. Assuming T_0 to be 450 K, measurements indicated in Ref. 13 (Fig. 23), which shows a maximum adiabatic wall temperature of 415.6 K, do not match measurements shown in other figures, which indicate a maximum adiabatic wall temperature of 432.2 K and a maximum recovery factor of 0.96. Similar remarks hold for the adiabatic wall temperature measurement in Ref. 15, which measure a peak wall temperature of about 448.3 K. This results in a recovery factor in excess of 0.97. It is true that the edge Mach number decreases in the presence of adverse pressure gradients, but that should not affect values of recovery factor, which is defined as

$$r = \frac{T_{\text{aw}} - T_e}{T_0 - T_e}$$

where

$$T_{\text{aw}} = T_e\left\{1 + [(\gamma - 1)/2]r M_e^2\right\}, \qquad T_0 = T_e\left\{1 + [(\gamma - 1)/2]M_e^2\right\}$$

Thus, without knowing the details of how the experiments were conducted, it is not obvious how one would explain these high values of the recovery factors.

Because the skin friction increases in the presence of adverse pressure gradients for hypersonic Mach numbers, use of a minimum skin-friction criterion to determine transition onset is not an option. Therefore, results presented in Figs. 4 and 5 employed the condition indicated in Eq. (18) to determine x_t.

Although transition was attributed in Refs. 13–15 to the growth of the second mode, we were unable to determine model constant b without inclusion of the contributions of the first mode. In other words, if the contribution of the first mode is set to zero, then values of b will either result in no transition or transition on the straight part of the cone.

Balakumar and Malik[33] showed that the growth rates for the first and second modes are comparable. This may explain the need to allow for contributions from both modes to arrive at a transition model. Further support to allow for contribution of the two modes follows from Refs. 13 and 14 (Figs. 23 and 2), which show that transition onset is not the same when the $M = 6$ tunnel is operated in conventional and quiet modes. If the second mode were the only mode contributing to transition, then one would expect that transition onset would be independent of intensity.

A mode constant $b = 0.054\,(6.5a)$ is used in the results indicated in Figs. 4 and 5. Both calculations employed a grid of 160 × 80 and

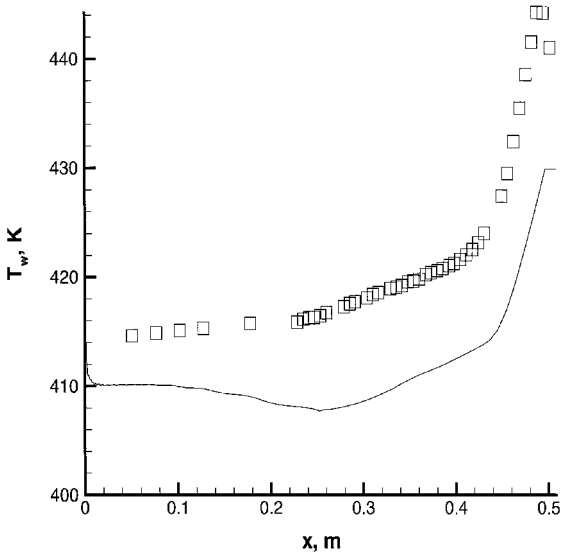

Fig. 4 Measured and computed adiabatic wall temperature: M = 6.0, 0.508-m (20-in.) flared cone.

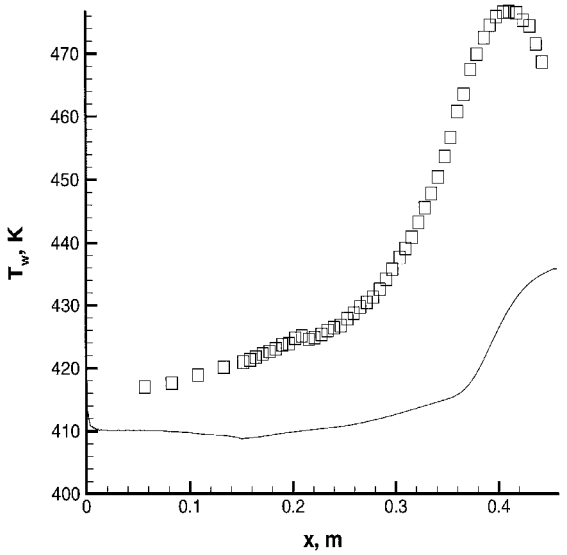

Fig. 5 Measured and computed adiabatic wall temperature: M = 6.0, 0.457-m (18-in.) flared cone.

an intensity $Tu = 0.1$. In spite of the difficulties associated with adiabatic wall temperature, excellent agreement with pressure measurements was obtained. Moreover, growth rate data in Ref. 15 suggest that transition onset starts at about $x = 0.356$ m (14 in.), which is consistent with current findings.

M = 8 Test Cases

Because a theory developed for natural transition is not suited to model experiments carried out in conventional wind tunnels, the goal here is to determine whether a transition mechanism based on the second mode will yield reasonable agreement with experiment. The value of intensity was not provided, and a value of 1.5% is assumed. It quickly became evident that such a goal cannot be achieved. In view of the results just indicated for $M = 6$ operation in quiet and conventional modes, this result is to be expected. Thus the model developed below allows for contributions from both modes. Because of the high levels of freestream intensities, a criterion based on Eq. (18) is meaningless. Thus, an onset criterion based on a minimum heat transfer is employed.

Two 7-deg half-angle cones were considered in this investigation: a straight cone and a flared cone characterized by $dP/dx = 4$. This corresponds to the nondimensional pressure gradient parameter $(L/C_{pc})(dC_p/dx)$, where L is the cone length and C_{pc} is the pressure coefficient on the conical forebody. For the two cases considered, $Re/L = 6.562 \times 10^6$/m, the length is 1.016 m (40 in.), and the wall temperature is 300 K. The grid employed is 201 × 91.

Figures 6 and 7 compare the predictions of the theory for sharp and flared cases. The model constants, which are obtained by numerical optimization, are $a = 0.016$ and $b = 0.01$. The scatter in the data (±10% accuracy) makes it difficult to determine the role of adverse pressure gradient on the measurements. Theory indicates an earlier transition of the flared case. Further, the underprediction of the heat flux in the turbulent region may be a result of the assumed intensity.

Both a and b are intensity dependent. Because the freestream intensity is in the bypass region, there is a question whether a general theory based on first- and second-mode scaling is viable for flows in conventional wind tunnels. More measurements at various intensities are needed to resolve this question.

Current results were obtained using a typical Navier–Stokes solver that employs a two-equation turbulence model. The cost of the calculation is comparable to that of a fully turbulent flow. This approach may be compared with e^n-methods, which require either Euler and boundary-layer codes or Navier–Stokes codes with higher resolution in order to generate accurate mean flows with smooth second derivatives, a stability code, specification of mode frequencies, and the value of n in order to determine transition onset. Moreover, the procedure has never been dynamically coupled to a flow solver to obtain the complete flowfield.

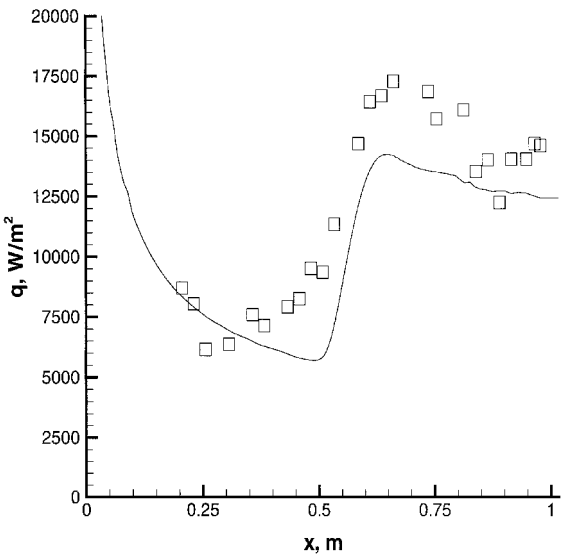

Fig. 6 Measured and computed heat flux: M = 8.0, straight cone.

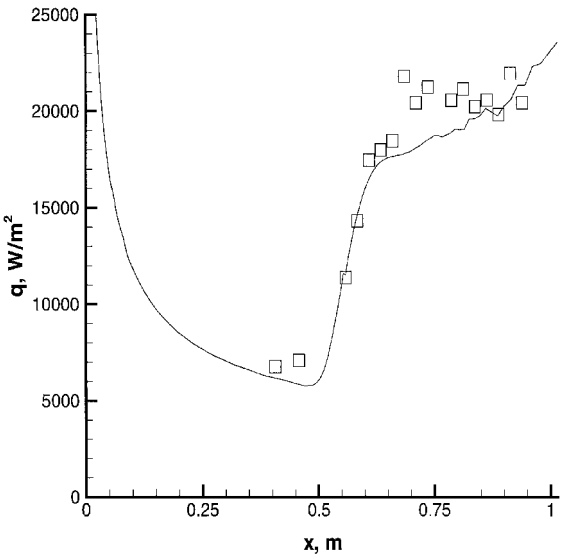

Fig. 7 Measured and computed heat flux: M = 8.0, flared cone.

Conclusion

The existing k–ζ transition/turbulent model developed by Warren and Hassan[1] can be extended to high Mach-number flows. Reasonable predictions of transition onset were indicated. Further, with the exception of the $M = 6$ data, fair to good agreement with experiment is obtained.

The work further confirms that treating transitional flows in a turbulence-like manner is a viable way of describing such flows.

The cost of calculating the entire flowfield is less than calculating the mean laminar flow in the e^n-method because stability codes require flow properties with smooth second derivatives, which necessitates a much finer grid.

Finally, there is a need to conduct experiments designed to determine onset and extent of transition in quiet tunnels. Such experiments would be of great use for code calibration and validation.

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